

Ghosts that Connect

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Abstract

The Faddeev–Popov procedure poses two conceptual puzzles. *Puzzle (1)*: if gauge-equivalent configurations represent the same physics, the quotient $\mathcal{A}/\mathcal{G}$ should suffice to compute physical amplitudes — yet the procedure requires anti-commuting auxiliary fields, the ghosts, with no analogue on the quotient. What structure of $\mathcal{A}$ do they encode? *Puzzle (2)*: gauge-fixing was supposed to eliminate local gauge symmetry, yet the gauge-fixed theory retains BRST — a residual symmetry that acts on the gauge potential as an infinitesimal gauge transformation. Why does it survive?

Both puzzles dissolve together. Following Dougherty (2021) and Dougherty and Read (2026), I take ghosts to encode classical content of $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$, but identify a different structure: a principal connection ϖ on this bundle. The ghost is ϖ ; the BRST operator is the vertical exterior derivative on field space; the Maurer–Cartan equation is its vertical Cartan structure equation. The Faddeev–Popov calculus draws only on ϖ ’s vertical content, which the algebraic reading also captures; ϖ ’s horizontal content, on which the Vilkovisky–DeWitt programme rests, supplies the cross-orbit pairing that gauge-fixing, dressing-based quantisation, and counterfactual comparison require and the quotient discards. BRST is the rigid, vertical symmetry that preserves this pairing; this is why it survives gauge-fixing.

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1 Introduction

By reputation, the Faddeev–Popov ghost is a trick. The path integral of a local gauge theory counts each physical configuration once per gauge transformation — an infinite overcounting — and the prescription of Faddeev and Popov (1967) tames it by selecting one representative per gauge-equivalence class. The price to pay for getting rid of the infinities is a pair of anti-commuting auxiliary fields, the *ghosts*, which enter the action and circulate in loops but never appear among the physical states. And the prescription does not quite finish the job: the gauge-fixed action retains a residual rigid nilpotent symmetry that mixes the gauge potential with the ghosts — the BRST symmetry of Becchi et al. (1976) and Tyutin (1975).

Taken at face value, the procedure poses two puzzles. *Puzzle (1)*: if gauge-equivalent configurations represent the same physics, the quotient $\mathcal{A}/\mathcal{G}$ — of the space $\mathcal{A}$ of gauge potentials by the gauge group $\mathcal{G}$ — ought to suffice for physical amplitudes; yet the ghosts have no analogue on the quotient. What is lost in passing from $\mathcal{A}$ to $\mathcal{A}/\mathcal{G}$? The standard reply is that $\mathcal{A}/\mathcal{G}$ is defined only as a set of equivalence classes with no intrinsic parametrisation, so one can’t help but work in $\mathcal{A}$. The reply is correct so far as it goes, but it leaves open whether the extra structure on $\mathcal{A}$ is a computational expedient or carries content of independent interest. *Puzzle (2)*: gauge-fixing was supposed to remove the infinite-dimensional redundancy of local gauge symmetry, yet the gauge-fixed theory admits a residual symmetry — BRST — that acts on the gauge potential exactly as an infinitesimal gauge transformation. What has gauge-fixing accomplished, if it has

neither passed to the quotient nor eliminated gauge symmetry? Both puzzles grow out of Faddeev–Popov folklore, and this paper will locate exactly where the folklore misfires.

The folklore in question, inherited from the Faddeev–Popov heuristic, treats the ghost as a mere calculational device for the functional determinant, with no standing outside the gauge-fixed path integral. Dougherty (2021) argues instead that ghosts encode classical geometric content of the theory’s configuration space — a central case of what Dougherty and Read (2026) systematise under the heading *non-positive quantisation*.¹ I take that conclusion as my starting point; the disagreement with Dougherty (2021) and Dougherty and Read (2026) is second-order, over the identity of the classical structure encoded. Their formulation is algebraic: ghosts are the degree-one generators of the Chevalley–Eilenberg complex of the action Lie algebroid of $\mathcal{G}$ on $\mathcal{A}$.² The algebraic presentation is correct but partial: it is the fibrewise record of a geometric structure that carries more content than the algebra can disclose. The business of this paper is to make that further content explicit.

The first extension addresses Puzzle (1). If one reads the BRST machinery as classical-content, the structure thrown away by the quotient is the Chevalley–Eilenberg complex. But this reading doesn’t say much about *why* that structure should be of independent physical or philosophical interest; i.e. it doesn’t say why throwing away that structure is problematic. The reason, I shall argue, is that while the quotient $\mathcal{A}/\mathcal{G}$ individuates gauge-equivalence classes, it cannot support the cross-orbit comparisons that quantisation and counterfactual reasoning both require. Summing amplitudes over distinct orbits, comparing local observables across configurations, and gluing regional subsystems into a global state all presuppose a cross-orbit pairing the quotient does not carry.³ §3.1 develops

¹*Non-positive quantisation* is Dougherty and Read (2026)’s umbrella term for quantisation procedures whose state space admits negative-norm vectors — ghosts are the paradigm case, alongside the indefinite-metric formalisms of Gupta–Bleuler and the like. Distinct from this is their *monist/pluralist* axis, which concerns not classical content but the relation between nominally different quantisation procedures — canonical, path-integral, Dirac-constrained-Hamiltonian, DeWitt–Faddeev–Popov, BRST, Batalin–Fradkin–Vilkovisky, Batalin–Vilkovisky. Monism, in Dougherty (2021)’s strong form, holds these to be ways of computing a single quantisation rather than rival quantisations; that thesis is orthogonal to the present paper, which fixes one prescription and asks what classical geometry its ghosts encode.

²The *action Lie algebroid* of $\mathcal{G}$ on $\mathcal{A}$ is the trivial vector bundle $\mathcal{A} \times \text{Lie } \mathcal{G}$ with anchor $\lambda \mapsto X_\lambda$, where X_λ is the orbit-tangent vector field generated by $\lambda \in \text{Lie } \mathcal{G}$. The bracket on sections, defined by the Lie bracket on constant sections and extended by the Leibniz rule, encodes the gauge action. Its *Chevalley–Eilenberg complex* is the cochain complex $\Lambda^\bullet(\text{Lie } \mathcal{G})^* \otimes C^\infty(\mathcal{A})$, with differential combining the bracket and the anchor; the degree-zero cohomology is the $\mathcal{G}$ -invariant functions on $\mathcal{A}$. See Dougherty and Read (2026, §3) for the full construction.

³The thought that the gauge-quotient leaves out structure has precedents: Healey (2007) on Aharonov–Bohm and the holonomy interpretation; Belot (2003) on the limits of treating symmetric models as identical. The modal-semantic parallel with counterpart-theoretic readings (Kment (2012), Roberts

that argument. In place of the quotient, gauge theory needs a connection on $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$. This sharpens a disagreement with the sophistication literature (Dewar, 2017) to which §6 returns.

The second extension addresses Puzzle (2). The cross-orbit pairing just mentioned can be given by a gauge-fixing. Read geometrically, gauge-fixing does not simply cut each orbit once but fixes a *counterpart relation* across orbits, pairing configurations on different orbits by designating which of them correspond (Gomes, 2025). At the infinitesimal level this counterpart relation is a connection on field space — a rule, at each configuration A , for separating directions along which only the gauge changes (the ‘vertical’ directions) from directions along which the physics changes (a complementary ‘horizontal’ set).⁴ On this reading, the ghost η is the connection. BRST’s survival of gauge-fixing then follows: BRST is the algebraic record of the vertical structure of $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$, and that fibre structure exists whether or not a section has been chosen. The standard Faddeev–Popov calculus draws only on vertical content; the gauge-invariant content of the Lagrangian concerns the horizontal data — where curvature and holonomy of the connection live, and which the Vilkovisky–DeWitt programme takes as primitive (§5.2).

In the non-Abelian cases covered by Singer’s theorem, the curvature of this connection cannot vanish globally: the theorem (Singer, 1978) rules out both a global gauge-fixing section and a globally flat connection for the bundles at issue — established for $M = S^4$ or S^3 with $G = \mathrm{SU}(N)$ — so any geometric account that extends globally must live with curvature. Here the two readings part ways. The algebraic reading, working from the Chevalley–Eilenberg differential, captures fibrewise content but cannot see the obstruction; the connection reading makes it visible, and shows that the counterpart relation generalises to path-dependent alignment via parallel transport — the field-theoretic realisation of Barbour’s best-matching programme (Barbour, 2003; Barbour and Bertotti, 1982; Gryb and Gomes, 2021).

The paper is organised as follows. §2 sets up the path-integral machinery and poses the puzzles in precise terms. §3 dissolves Puzzle (1) philosophically and Puzzle (2) operationally: gauge-fixing fixes a counterpart relation, and BRST’s verticality and rigidity together preserve it. §4 delivers the geometric reconstruction, identifying the ghost with the functional connection, the BRST operator with the leafwise exterior derivative along orbits, and showing that the connection is non-flat by the Gribov–Singer theorem.

(2026)) is taken up in §3.1. Gomes (2025, 2026) develops that gauge-theoretic version.

⁴The formal object is a principal connection ϖ on the bundle $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$ in the sense of Kobayashi–Nomizu, transposed from finite-dimensional bundles on spacetime to the infinite-dimensional bundle of field space. §4 gives the construction; for the original treatment, Singer (1978); for the rigorous modern mathematical treatment, Diez and Rudolph (2019); for the complete bibliography, Gomes and Riello (2017).

§5 answers the conventionality objection and locates the reading relative to the Vilkovisky–DeWitt programme. §6 returns to the two puzzles, folds in the best-matching reading, and locates the paper in the debate over sophistication about symmetry (Dewar, 2017; Gomes, 2026). The technical (book-keeping) aspects of the antighost and Nakanishi–Lautrup field are collected in Appendix A.⁵

2 The path integral, gauge-fixing, and BRST

2.1 The problem and the remedy

Here we fix a Yang–Mills theory on a compact Riemannian four-manifold M with gauge group G (connected, compact, semisimple) on a principal bundle $P \rightarrow M$. Field space is denoted by $\mathcal{A}$, and it is the affine space of G -connections on P . The gauge group $\mathcal{G}$ — the group of vertical bundle automorphisms of P — acts on $\mathcal{A}$ by pullback, with quotient $\mathcal{A}/\mathcal{G}$. The Yang–Mills action $S[A] = -\frac{1}{2}\int_M \text{tr}(F \wedge \star F)^6$ is $\mathcal{G}$ -invariant, so the naive path integral

$$Z = \int_{\mathcal{A}} \mathcal{D}A e^{-S[A]} \quad (1)$$

assigns each physical configuration with a multiplicity equal to the volume of $\mathcal{G}$, which is infinite.

The infinite volume is, by itself, a manageable problem, for in normalised amplitudes

$$\langle \mathcal{O} \rangle = \frac{\int_{\mathcal{A}} \mathcal{D}A \mathcal{O}[A] e^{-S[A]}}{\int_{\mathcal{A}} \mathcal{D}A e^{-S[A]}} \quad (2)$$

$\text{vol}(\mathcal{G})$ appears as an overall factor in numerator and denominator and cancels. The cancellation is not enough, however: the same redundancy reappears at the next order, in the guise of the Hessian’s degenerate kernel. At a solution A_0 , $\mathcal{G}$ -invariance implies that $H[A_0] \cdot D_{A_0} \lambda = 0$ for every $\mathfrak{g}$ -valued function λ , where $H[A_0] := \delta^2 S / \delta A^2|_{A_0}$ is the Hessian: every orbit-tangent direction is a zero mode, so the kernel is infinite-dimensional. Integrating the Gaussian $e^{-\frac{1}{2}\delta A \cdot H \cdot \delta A}$ over the zero modes of $H[A_0]$ recovers $\text{vol}(\mathcal{G})$ as the unsuppressed direction, so that at one loop $\det H = 0$ and the volume divergence reappears in the partition function. Thus the propagator, being the inverse of H , does not exist, and perturbation theory cannot begin. Passing to the physical quotient $\mathcal{A}/\mathcal{G}$

⁵Anomalies, the Batalin–Vilkovisky extension of BRST, and the role of ϖ at spatial boundaries lie outside the scope of this paper. For anomalies in the geometric BRST framework, see Bertlmann (2000). For BV/BFV, see Batalin and Vilkovisky (1981); Cattaneo et al. (2014). For the functional connection at boundaries, see Carrozza and Höhn (2022); Donnelly and Freidel (2016); Gomes et al. (2019).

⁶With anti-Hermitian generators tr is negative-definite, and $F \wedge \star F = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \text{vol}_g$, so the prefactor reproduces the standard $\frac{1}{4} \int_M F_{\mu\nu}^a F^{a\mu\nu}$.

would cure both problems, but $\mathcal{A}/\mathcal{G}$ is defined only as a set of equivalence classes with no intrinsic parametrisation, so any computation must operate in $\mathcal{A}$.

Gauge-fixing is the intermediate resolution. A *gauge-fixing functional* is a map $F : \mathcal{A} \rightarrow W$, with W a vector space modelled on $\text{Lie } \mathcal{G}$, chosen to impose one condition per gauge direction. The zero set $\Sigma_F := F^{-1}(0) \subset \mathcal{A}$ is a submanifold of field space. On a sufficiently small neighbourhood where the restriction of $\delta F/\delta A$ to orbit directions is invertible, Σ_F meets nearby orbits transversally and once.⁷ On such a region Σ_F gives a local chart on $\mathcal{A}/\mathcal{G}$, and, with $A^\sigma \in \Sigma_F$ being the representative on the section and $g \in \mathcal{G}$ the unique element carrying A^σ to A , the assignment $A \mapsto (A^\sigma, g)$ yields a local trivialisation $\mathcal{A} \simeq \Sigma_F \times \mathcal{G}$. Geometrically, gauge-fixing amounts to a choice of complement to the orbit directions (see Figure 1):

$$T_A \mathcal{A} = V_A \oplus H_A, \quad (3)$$

where $V_A = \{D_A \lambda : \lambda \in \Omega^0(M, \mathfrak{g})\}$ is the vertical (orbit-tangent) subspace and $H_A = \ker(\delta F/\delta A)|_A$ is the (integrable) horizontal complement determined by F : at points of Σ_F it is the tangent space to the section, and elsewhere the tangent space to the level set $F^{-1}(F[A])$.⁸ The vertical summand is intrinsic to the bundle structure; the horizontal summand depends on the choice of F . In §4 I will generalise this splitting to cases in which no global section exists.

Changing variables from A to (A^σ, g) , the measure factorises as $\mathcal{D}A = \mathcal{D}A^\sigma \cdot \mathcal{D}g \cdot J[A^\sigma]$, with J the Jacobian of the decomposition (3). $\mathcal{G}$ -invariance of the action lets the $\mathcal{D}g$ -integral factor out as $\text{vol}(\mathcal{G})$, which cancels in normalised amplitudes, leaving an integral over Σ_F weighted by J . Within a patch where the sign of the determinant is fixed — as in the first Gribov region, where $\mathcal{M}[A]$ is positive — the Jacobian is represented by the determinant of the Faddeev–Popov operator,

$$\mathcal{M}[A] := \left. \frac{\delta F[A^\lambda]}{\delta \lambda} \right|_{\lambda=0}, \quad (4)$$

where A^λ is the gauge transform of A by $\lambda \in \Omega^0(M, \mathfrak{g})$. Geometrically, $\mathcal{M}[A]$ relates variations along the orbit through A to variations of the gauge-fixing condition; $\det \mathcal{M}[A]$ is accordingly the Jacobian of the projection from orbit-tangent directions onto W , evaluated at the unique point where the orbit through A meets the section (Babelon and Viallet,

⁷The restriction to such a region is unavoidable: no gauge-fixing section extends across the whole bundle. See Gribov (1978) for the original observation in Coulomb gauge, and §4.2 for the global argument.

⁸The distribution $\ker(\delta F/\delta A)$ is in general not $\mathcal{G}$ -equivariant — in Lorenz gauge it is $\{X : \partial^\mu X_\mu = 0\}$ at every A , a condition the adjoint action does not preserve — so it is not itself a principal connection in the sense of §4. The connection a gauge-fixing determines is obtained by restricting $\ker(\delta F/\delta A)$ to Σ_F and propagating it over each orbit by the group action: the result is equivariant by construction, flat, and has the $\mathcal{G}$ -translates of Σ_F as its horizontal leaves.

1979). Writing a determinant in the exponent calls for anti-commuting variables; the Berezin identity

$$\det \mathcal{M}[A] = \int \mathcal{D}\eta \mathcal{D}\bar{\eta} \exp\left(-\int_M \bar{\eta} \cdot \mathcal{M}[A] \cdot \eta\right) \quad (5)$$

turns the Jacobian into a term in the action, and the path integral becomes

$$Z = \int_{\mathcal{A}} \mathcal{D}A \delta(F[A]) \int \mathcal{D}\eta \mathcal{D}\bar{\eta} e^{-S[A]-S_{\text{FP}}[A,\eta,\bar{\eta}]}, \quad (6)$$

with $S_{\text{FP}}[A,\eta,\bar{\eta}] = \int_M \bar{\eta} \cdot \mathcal{M}[A] \cdot \eta$. The fields η and $\bar{\eta}$ are the *ghost* and *antighost*. On the Faddeev–Popov reading they are mere calculational tools: neither physical fields nor asymptotic states, present only because a Grassmann algebra allows the determinant to be written as a functional integral of a local quantity. In the Abelian case $\mathcal{M}[A] = \mathcal{M}$, so the Jacobian is field-independent and factors out alongside the volume of the gauge group; in the non-Abelian case it cannot. §4 returns to these objects and gives them a geometric reading.⁹

2.2 Puzzles about BRST symmetry

The total action $\tilde{S}$ is no longer gauge-invariant, since S_{FP} and S_{GF} both depend on the gauge-fixing functional F . It does, however, admit a residual symmetry. With a single anti-commuting parameter ε , define

$$sA_\mu = D_\mu \eta, \quad s\eta = -\frac{1}{2}[\eta, \eta], \quad s\bar{\eta} = iB, \quad sB = 0, \quad (8)$$

and extend s to products by the graded Leibniz rule, with ghost-number grade $+1$.¹⁰ Then $s\tilde{S} = 0$ and $s^2 = 0$ on every field. The operator s is the BRST operator of Becchi et al. (1976) and Tyutin (1975).

Three features of (8) are important for what follows. First, the action on A_μ is that of an infinitesimal gauge transformation with parameter η : BRST turns the ghost into

⁹The δ -function on Σ_F is inconvenient in a perturbative expansion. The standard move is to trade it for a Gaussian peak around Σ_F , by introducing an auxiliary bosonic field B (the Nakanishi–Lautrup field) and writing the gauge-fixing condition as an exponential weight:

$$S_{\text{GF}}[A, B] = \int_M \left(B \cdot F[A] - \frac{\xi}{2} B \cdot B \right). \quad (7)$$

The total gauge-fixed action is $\tilde{S} = S + S_{\text{FP}} + S_{\text{GF}}$, and the path integral runs unconstrained over $\mathcal{A}$, the ghost fields, and the Nakanishi–Lautrup field. The Gaussian width ξ is a gauge parameter; physical predictions are independent of it. See Peskin and Schroeder (1995), §16.4. The field B is integrated along the imaginary axis (equivalently $B \mapsto iB$), so that $\int \mathcal{D}B e^{-S_{\text{GF}}}$ is a convergent Gaussian yielding the standard quadratic gauge-fixing term $\frac{1}{2\xi} \int_M F[A] \cdot F[A]$; the appendix carries the i -conventions explicitly.

¹⁰ ε enters multiplicatively: every BRST transformation of a field ϕ is $\delta_\varepsilon \phi = \varepsilon s\phi$, with ε suppressed in the displayed conventions (8) and understood throughout. The operator s marks the BRST direction; ε is the (rigid) parameter scaling the motion.

a stand-in for a gauge parameter. Second, the action on the ghost, $s\eta = -\frac{1}{2}[\eta, \eta]$, has the shape of a Maurer–Cartan equation.¹¹ Third, the parameter ε is a single Grassmann scalar — not a function on M or a functional on $\mathcal{A}$. BRST is *rigid*: ε has no spacetime or configuration dependence. The ghost η is not the parameter of the transformation but one of its ingredients: a $\mathfrak{g}$ -valued Grassmann field on spacetime, indistinguishable in type from an anti-commuting gauge parameter, but a field of the gauge-fixed theory in its own right. The puzzles below turn on keeping ε and η apart.

The setup sharpens both puzzles. For Puzzle (1), the Gribov–Singer theorem turns the global form of the obstacle into a topological statement: no smooth section $\sigma : \mathcal{A}/\mathcal{G} \rightarrow \mathcal{A}$ extends globally for the bundles at issue (Singer, 1978, Theorem 7). The impossibility of a global section does not yet say what gauge-fixing discards locally, where a section exists; whether the procedure supplies structure beyond orbit-individuation — and in what form — is the question §3 and §4 take up. For Puzzle (2), the splitting (3) implies $H[A_0]$ is non-degenerate on H_{A_0} ¹² and vanishes on V_{A_0} ; S_{GF} contributes a quadratic form non-degenerate on V_{A_0} ; together they are invertible across $T_{A_0}\mathcal{A} = V_{A_0} \oplus H_{A_0}$, so the Hessian of $\tilde{S}$ exists and so does the propagator. Yet $sA_\mu = D_\mu\eta$ remains: BRST acts on A_μ exactly as an infinitesimal gauge transformation. The worry is not whether some orbit-tangent direction survives (one does), but whether an infinite-dimensional family of them does, since a $\lambda(x)$ ’s worth of zero modes would resurrect the original kernel.

3 Rigidity and counterparts

If gauge-fixing has neither fully eliminated gauge symmetry nor passed to the physical quotient, what has it accomplished? One idea answers both puzzles: gauge-fixing fixes a *counterpart relation* across orbits — a between-orbit pairing that the quotient $\mathcal{A}/\mathcal{G}$ discards, and that BRST, contrary to first appearances, preserves. Two features of BRST conspire to this end. *Verticality*: the field-space one-form $sA = D_A\eta$ takes values in orbit-tangent (vertical) vectors. *Rigidity*: η is a single odd field, not a configuration-dependent parameter $\lambda[A]$, so the BRST flow acts uniformly across $\mathcal{A}$. Together they leave the cross-orbit pairing intact.

¹¹For a Lie group H with Lie algebra $\mathfrak{h}$, the canonical left-invariant $\mathfrak{h}$ -valued one-form θ on H satisfies $d\theta + \frac{1}{2}[\theta, \theta] = 0$. Why the BRST algebra on the ghost takes this shape is the question §4.1 answers.

¹²Generically: at backgrounds with moduli — instantons are the standard case — $H[A_0]$ retains finitely many zero modes on H_{A_0} , handled by collective coordinates rather than gauge-fixing.

3.1 The counterpart relation

On the classical-content reading of Dougherty (2021) and Dougherty and Read (2026), ghosts encode classical structure on $\mathcal{A}$; their algebraic identification is with the Chevalley–Eilenberg complex. As we will see, that classical structure is the machinery of cross-orbit alignment that the quotient discards and the path integral uses.

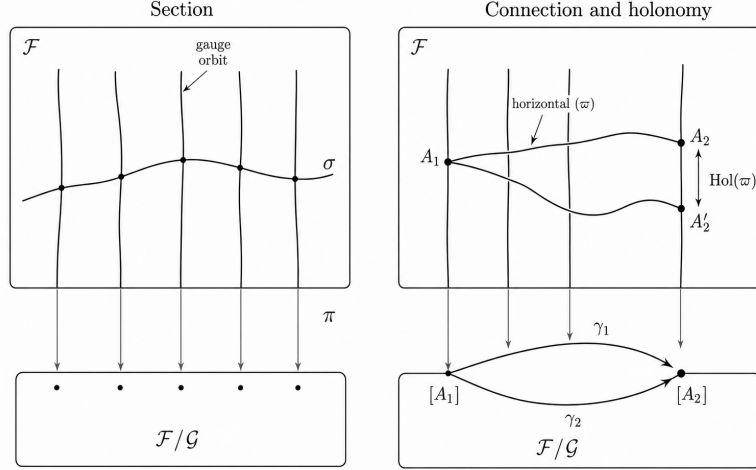


Figure 1: Left: a gauge-fixing section σ cuts each orbit once, picking one representative per fibre. Configurations $A_1 \in [A_1]$ and $A_2 \in [A_2]$ designated by σ are *counterparts relative to* σ : a between-orbit pairing additional to the within-orbit equivalence individuated by the quotient. Right: in the absence of a global section, alignment is supplied by the horizontal lifts of a connection ϖ . Different paths $\gamma_1, \gamma_2 \subset \mathcal{A}/\mathcal{G}$ from $[A_1]$ to $[A_2]$ produce horizontal lifts ending at points A_2 and A'_2 on the same fibre; their mismatch is the holonomy $\text{Hol}(\varpi)$. The right panel previews §4.

Consider two orbits $[A_1], [A_2] \in \mathcal{A}/\mathcal{G}$. The section picks out one representative on each, $\sigma([A_1]) \in [A_1]$ and $\sigma([A_2]) \in [A_2]$, pairing them as the canonical orbit-representatives for Σ_F . The quotient alone does not do this: it knows *that* two configurations are gauge-equivalent but not *how* — not which gauge transformation carries one to the other (Dougherty, 2021) — and it knows nothing at all about the correspondence between configurations on *different* orbits. A section, by contrast, answers the question “given a configuration on orbit $[A_1]$, which configuration on orbit $[A_2]$ is its counterpart?” Different sections give different answers.

Any quantisation summing contributions from distinct orbits uses such a pairing.¹³ The Faddeev–Popov Jacobian itself is an infinitesimal cross-orbit quantity (§2.1). In spacetime

¹³Not every quantisation does sum over configurations. Locally covariant quantum field theory (Brunetti et al., 2003) builds observables functorially — an algebra $\mathcal{A}(M)$ for each admissible background M , with injective morphisms arising from isometric embeddings $M \hookrightarrow N$ — and involves no path integral over

theories with diffeomorphism symmetry, comparing local observables at a spacetime point across configurations, and gluing regional subsystems into a global state, both presuppose an alignment between configurations on different orbits (Gomes, 2026).

This cross-orbit alignment admits a natural reading as a *counterpart relation* in the sense familiar from modal semantics. For instance, Kment (2012) argues that if an anti-haecceitist semantics for chance and counterfactuals is to underwrite the relevant modal distinctions, bare qualitatively individuated worlds—here the orbits in $\mathcal{A}/\mathcal{G}$ —are insufficient: the semantics must be enriched by world descriptions involving counterpart mappings, or by a generalised similarity ordering over individuals. Roberts (2026) develops a related move in higher-order modal language, explicitly without counterpart theory: ordinary modal discourse implicitly operates with selector-like haecceitistic structure, and selectors play the structural role that counterpart functions play for Kment. Both frameworks require more than qualitative individuation of possibilities: they require a structural pairing across them. The gauge-theoretic version of this thought (Gomes, 2025, 2026) is that the quotient and the counterpart relation do different jobs: the quotient individuates orbits; the counterpart relation aligns points across orbits; and both amplitude summation and counterfactual reasoning turn on the latter as well as the former.¹⁴ Other descriptions of what gauge-fixing does are available; the case for this one is the economy argument of §5.

The construction, following Gomes (2025), runs as follows. The *dressing* associated with Σ_F is the map $g_\sigma : \mathcal{A} \rightarrow \mathcal{G}$ sending each configuration A to the unique gauge transformation carrying it to its representative on Σ_F :

$$F[A^{g_\sigma(A)}] = 0. \quad (9)$$

Uniqueness holds within the Gribov region, where the gauge-fixing is non-degenerate. The right action of $\mathcal{G}$ shifts the dressing equivariantly: applying $h \in \mathcal{G}$ to A moves the slice-aligning transformation by h^{-1} on the left,

$$g_\sigma(A^h) = h^{-1} g_\sigma(A), \quad (10)$$

as one reads off uniquely from $F[(A^h)^{h^{-1}g_\sigma(A)}] = F[A^{g_\sigma(A)}] = 0$. Two configurations A_1 and A_2 are *counterparts relative to σ* when the same gauge transformation dresses each to its representative on Σ_F :

$$g_\sigma(A_1) = g_\sigma(A_2). \quad (11)$$

field configurations to weight or sum. Its covariance is functorial naturality across spacetime backgrounds: a different relation from a pairing across gauge orbits on a fixed background, and a more restricted one, holding only when an admissible embedding exists. The pairing at issue here arises for quantisations that do sum over orbits: the Lagrangian path integral and its dressing-based variants.

¹⁴The argument does not require haecceitism in the sense of primitive individuation — only the claim that *some* non-qualitative pairing is required, whether underwritten metaphysically or semantically.

This is an equivalence relation on $\mathcal{A}$. Within an orbit it is trivial: equivariance (10) gives $g_\sigma(A^h) = h^{-1}g_\sigma(A)$, so equality of dressings on the same orbit forces $h = \text{id}$. The non-trivial content of counterparthood is therefore strictly between orbits. The associated structural object is the *equilocality relation*¹⁵

$$\mathcal{E}_\sigma : \mathcal{A} \times \mathcal{A} \rightarrow \mathcal{G}, \quad (A_1, A_2) \mapsto g_\sigma(A_1)^{-1} g_\sigma(A_2), \quad (12)$$

which describes the relative dressing of A_1 and A_2 and is the identity precisely when they are counterparts: $\mathcal{E}_\sigma(A_1, A_2) = \text{id}$ iff $g_\sigma(A_1) = g_\sigma(A_2)$. The same object captures within-orbit relations: for $A_2 = A_1^h$ on the same orbit, equivariance gives $\mathcal{E}_\sigma(A_1, A_1^h) = g_\sigma(A_1)^{-1} h^{-1} g_\sigma(A_1)$, the dressing-conjugate of h^{-1} — so $\mathcal{E}_\sigma$ encodes within-orbit gauge equivalence as well as cross-orbit alignment. Equation (12) is translation-invariant, since

$$\mathcal{E}_\sigma(A_1^h, A_2^h) = (h^{-1}g_\sigma(A_1))^{-1} h^{-1}g_\sigma(A_2) = g_\sigma(A_1)^{-1} h h^{-1} g_\sigma(A_2) = \mathcal{E}_\sigma(A_1, A_2). \quad (13)$$

Chain composition is automatic from the group structure:

$$\mathcal{E}_\sigma(A_1, A_2) \mathcal{E}_\sigma(A_2, A_3) = \mathcal{E}_\sigma(A_1, A_3), \quad (14)$$

which ensures that counterparthood threads consistently across triples of orbits.

Of course, different gauge-fixings yield different counterpart relations; Coulomb gauge picks out a different alignment than Lorenz gauge.

One might hope to sidestep the counterpart thesis by quantising in terms of relational observables, anchoring to asymptotic boundary data, or passing to Dirac-dressed variables. These are not, however, alternatives to cross-orbit alignment; they are ways of fixing it. Dirac dressings realise the flat case of a functional connection; edge-mode and asymptotic prescriptions fix alignment through boundary data; relational-observable constructions dress gauge-variant fields into invariant ones in the same manner.¹⁶

3.2 BRST preserves counterparts

Recall the worry from §2.2: BRST acts on A_μ as an infinitesimal gauge transformation, and a function-parametrised gauge symmetry would resurrect the Hessian kernel that gauge-fixing was meant to remove. The original zero modes of $H[A_0]$ were *bosonic* tangent vectors $D_{A_0}\lambda$, parametrised by a function $\lambda \in \Omega^0(M, \mathfrak{g})$; the gauge-fixing term modified the quadratic form so that those directions leave the kernel. BRST produces an orbit-tangent direction $sA = D_A\eta$ of the same shape, but η is not free: it is a single Grassmann field of

¹⁵Written ε_σ in Gomes (2025); I write $\mathcal{E}_\sigma$ to keep it apart from the BRST Grassmann parameter ε of §2.2.

¹⁶For Dirac dressings as the flat case, Gomes et al. (2019). For edge modes, Carrozza and Höhn (2022); Donnelly and Freidel (2016). For dynamical reference frames, Goeller et al. (2022).

the gauge-fixed theory. The gauge-fixing apparatus is invariant not under a restored local gauge action, but only under the rigid BRST differential.¹⁷

Two features of BRST preserve the counterpart relation. The first is *verticality*: $sA = D_A\eta$ lands in the orbit-tangent (vertical) directions, with η as the orbit-shift parameter. §4.1 gives η 's geometric type; for the present argument it suffices that sA never leaves V_A . The second is *rigidity*: η is a single odd field of the gauge-fixed theory, not a configuration-dependent parameter $\lambda[A]$ chosen separately at each A . So one and the same η enters the transformation at A_1 and A_2 , and the BRST flow acts uniformly across $\mathcal{A}$. ‘Rigidity’ here extends the scalarity of ε from §2.2 to the configuration-independence of η as an integration field; the cross-orbit argument that follows turns on the latter, not the former.

Rigidity preserves counterparts. The dressing equivariance (10) with $h = \exp(\varepsilon\eta)$ gives, at first order in the Grassmann parameter ε ,

$$s g_\sigma(A) = -\eta g_\sigma(A), \quad s g_\sigma(A)^{-1} = g_\sigma(A)^{-1} \eta. \quad (15)$$

The graded Leibniz rule applied to (12) then yields

$$s \mathcal{E}_\sigma(A_1, A_2) = g_\sigma(A_1)^{-1} \eta g_\sigma(A_2) - g_\sigma(A_1)^{-1} \eta g_\sigma(A_2) = 0. \quad (16)$$

The equilocality relation is BRST-closed; in particular, the counterpart condition $\mathcal{E}_\sigma = \text{id}$ is preserved. The computation applies s to a function of two configurations, and so presupposes that one and the same η transforms both — the diagonal extension of the BRST action to $\mathcal{A} \times \mathcal{A}$. That extension is where rigidity enters the formalism; the cancellation in (16) is its direct expression.¹⁸ A field-dependent shift $A \mapsto A^{\exp(\lambda[A](x))}$, vertical but not rigid, would not preserve counterparts: λ would differ at A_1 and A_2 , and the cancellation in (16) would fail. A rigid but non-vertical shift would disturb the alignment between orbits. Verticality and rigidity are both required.

How rigid must the parameter be? Translation by a fixed $f \in \text{Lie } \mathcal{G}$ preserves counterparts — this is the invariance (13) in infinitesimal form. But BRST-closure of a field-dependent parameter ($sf = 0$: f constant along each orbit) is too weak: such an f can still differ from orbit to orbit, hence at A_1 and A_2 , and the cancellation in (16) fails. The surviving freedom is exactly a shift of the gauge condition by a field-independent gauge

¹⁷“Rigid” does not mean “field-independent” in a naive sense. The transformation $sA = D_A\eta$ depends on A through the covariant derivative and on the ghost field η . The free Grassmann parameter ε multiplying s , by contrast, is the same scalar at every point of M and every point of $\mathcal{A}$ — a function’s worth of parameters has been withheld.

¹⁸The argument extends to the path-dependent generalisation of §4: if A_2 is the endpoint of a horizontal lift of a path through A_1 , the uniform shift carries A_2 to the endpoint of the horizontal lift through $A_1^{\exp(\varepsilon\eta)}$.

parameter, which conjugates $\mathcal{E}_\sigma$ by a constant and so leaves the counterpart condition $\mathcal{E}_\sigma = \text{id}$ intact.

Two questions remain for §4. First, why does the BRST operator satisfy $s\eta = -\frac{1}{2}[\eta, \eta]$? The Maurer–Cartan shape is not explained by rigidity alone; rigidity says only that the parameter is a scalar. Second, how does the counterpart relation behave globally? Within a Gribov region a gauge-fixing supplies one, but globally the Gribov–Singer theorem (Singer, 1978, Theorem 7) forbids any continuous section $\sigma : \mathcal{A}/\mathcal{G} \rightarrow \mathcal{A}$ for the bundles at issue, so no global cross-orbit identification of the form (11) exists. Both questions have the same answer: η is a principal connection on the bundle of field space, the Maurer–Cartan equation is the vertical Cartan structure equation, and parallel transport along the connection extends the counterpart relation across $\mathcal{A}$ even where no section does. §4 makes this precise.

4 Ghosts as the connection on field space

Both questions from §3 — why the BRST algebra takes Maurer–Cartan shape, and what replaces the counterpart relation when no global section exists — share an answer. The underlying structure is familiar from finite-dimensional differential geometry: a principal bundle with connection. The bundle in question is not a bundle over spacetime but the infinite-dimensional bundle $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$ of gauge potentials over physical configurations (Diez and Rudolph, 2019). The ghost is its connection one-form, and the BRST algebra falls out of the defining properties of any principal connection on any principal bundle.

4.1 Field space as a principal bundle

The gauge group $\mathcal{G}$ acts freely and properly on field space $\mathcal{A}$ away from reducible configurations.¹⁹ On that stratum, $\mathcal{A}$ is a principal $\mathcal{G}$ -bundle,

$$\pi : \mathcal{A} \longrightarrow \mathcal{A}/\mathcal{G}, \tag{17}$$

with vertical tangent subspace $V_A = \{D_A \lambda : \lambda \in \Omega^0(M, \mathfrak{g})\}$ at each A and structure group $\mathcal{G}$ acting on fibres by pullback. The vertical distribution is intrinsic to the gauge action: V_A is the tangent space to the orbit through A , requiring no further choice. What does

¹⁹A configuration A is *reducible* when its stabiliser in $\mathcal{G}$ is non-trivial. Reducible configurations form a meagre set in the Sobolev topologies used to topologise $\mathcal{A}$; the generic configuration has trivial stabiliser (Kondracki and Rogulski, 1983; Mitter and Viallet, 1981). On the reducible locus the bundle structure is replaced by a stratification handled by slice theorems (Diez and Rudolph, 2019; Ebin, 1970; Isenberg and Marsden, 1982). I work on the open dense stratum of irreducible configurations throughout.

require a choice is the *horizontal complement* — a rule, at each A , for separating directions along which only the gauge changes from those along which the physics changes.

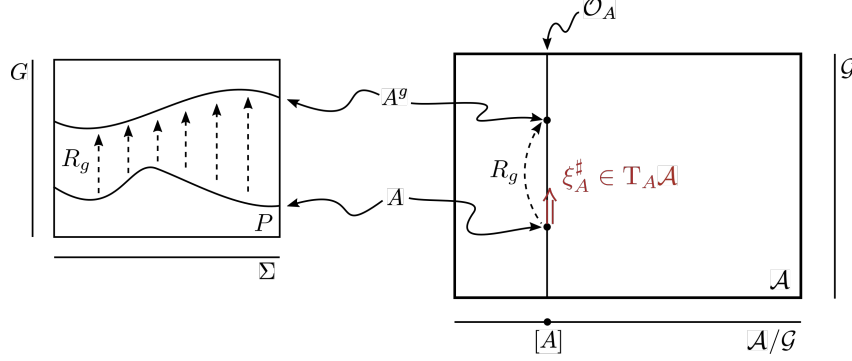


Figure 2: The principal bundle of field space (right) by analogy with the finite-dimensional principal bundle $P \rightarrow M$ over spacetime (left). The fibre $\mathcal{O}_A$ at $[A] \in \mathcal{A}/\mathcal{G}$ is the gauge orbit through A ; the structure group $\mathcal{G}$ acts by pullback, with right action R_g . A vertical tangent vector $\xi_A^\# \in T_A\mathcal{A}$ is the orbit-tangent vector generated by $\xi \in \text{Lie } \mathcal{G}$ via the fundamental-field map $\#_A : \text{Lie } \mathcal{G} \rightarrow V_A$, $\xi \mapsto \xi_A^\# = D_A\xi$.

Recall that a connection on a finite-dimensional principal G -bundle $P \rightarrow M$ is a $\mathfrak{g}$ -valued one-form $\omega \in \Omega^1(P, \mathfrak{g})$ restricting to the identity on fundamental vertical fields and G -equivariant under the adjoint action; its kernel is the horizontal distribution, and its curvature $\Omega = d\omega + \frac{1}{2}[\omega, \omega]$ measures the failure of that distribution to integrate. A *functional connection* on $\mathcal{A}$ is the infinite-dimensional analogue (Babelon and Viallet, 1981; Gomes and Riello, 2017; Singer, 1978): a one-form $\varpi \in \Omega^1(\mathcal{A}, \text{Lie } \mathcal{G})$ satisfying

$$\varpi(X^\#) = X \quad \text{for every } X \in \text{Lie } \mathcal{G}, \quad (18)$$

$$R_g^*\varpi = \text{Ad}_{g^{-1}}\varpi \quad \text{for every } g \in \mathcal{G}. \quad (19)$$

The kernel of ϖ defines a horizontal distribution $H_A = \ker \varpi|_A$ complementary to V_A . This is the splitting (3) again,

$$T_A\mathcal{A} = V_A \oplus H_A, \quad (20)$$

but now fixed intrinsically by ϖ rather than by a choice of gauge-fixing functional F . The vertical summand is the same in both. The horizontal summand here is gauge-equivariant by construction ((19)).

The choice of ϖ is the choice of horizontal complement. The defining equations (18)–(19) are intrinsic to the bundle and make no reference to a gauge-fixing functional F . When ϖ is flat, the two presentations of the horizontal complement — by a section and by a connection — coincide locally, and ϖ adds nothing the section did not already supply; §4.2 spells out the equivalence. When ϖ cannot be made globally flat, the connection extends globally where no section does, and the Gribov–Singer theorem rules out global

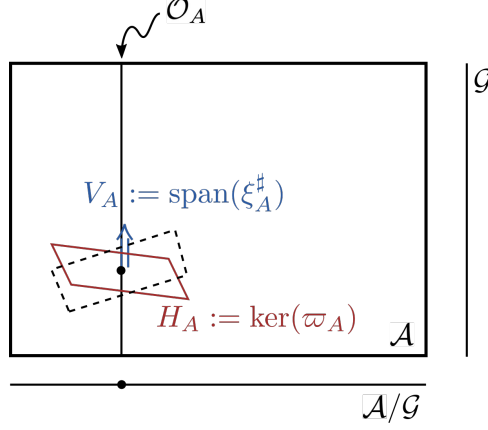


Figure 3: A functional connection ϖ specifies a horizontal complement $H_A = \ker(\varpi_A)$ to the vertical subspace $V_A = \text{span}(\xi_A^\#)$ at each $A \in \mathcal{A}$. The horizontal subspace varies with A ; its variation is encoded in the curvature $\mathfrak{F}$ of ϖ .

flatness in non-Abelian Yang–Mills, leaving the connection presentation as the only one that survives.

The curvature of ϖ is

$$\mathfrak{F} := \delta\varpi + \frac{1}{2}[\varpi, \varpi], \quad (21)$$

with δ the exterior derivative on $\mathcal{A}$ and $[\cdot, \cdot]$ the graded bracket on Lie $\mathcal{G}$ -valued forms.²⁰ $\mathfrak{F}$ is horizontal by construction: it vanishes whenever any argument is vertical. The BRST algebra is the *vertical* content of the structure equation relating $\delta\varpi$ to $\mathfrak{F}$, and that vertical content is the same whether $\mathfrak{F}$ vanishes or not.

Now we can make two identifications. The BRST operator is the leafwise exterior derivative along the gauge orbits,

$$s \leftrightarrow \delta_V, \quad (22)$$

and the ghost is the connection,

$$\eta \leftrightarrow \varpi. \quad (23)$$

Two senses of ‘vertical’ should be distinguished. In the intrinsic, leafwise sense, δ is evaluated only on vertical arguments. The vertical distribution $\mathcal{V} \subset T\mathcal{A}$ is intrinsic to the gauge action — $V_A = \{D_A\lambda\}$ exists whether or not a connection is present — and the resulting fibrewise exterior derivative on $\Gamma(\Lambda^\bullet \mathcal{V}^*)$ is the standard leafwise de Rham differential of the orbit foliation, requiring no further structure. A stronger decomposition splits δ on all of $\Omega^\bullet(\mathcal{A})$ as $\delta = \delta_V + \delta_H$, with δ_H acting on horizontal arguments; that decomposition requires a horizontal projector, equivalently a connection. The BRST equations (8) require only the first sense; i.e. it only needs evaluation on the vertical

²⁰Some authors use $\mathfrak{F} = \delta\varpi - \frac{1}{2}[\varpi, \varpi]$; the sign is absorbed into the bracket convention, and nothing below depends on the choice. I follow Gomes and Riello (2017).

arguments. The second, broader application enters when one wants δ_V to act on non-vertical arguments.

The identification (23) relates two objects that at first sight live in different places: the ghost $\eta(x)$ is a $\mathfrak{g}$ -valued Grassmann function on spacetime, while ϖ is a Lie $\mathcal{G}$ -valued one-form on field space. The two are matched by the Lie algebra of the gauge group itself. Since $\mathcal{G}$ is the group of vertical bundle automorphisms of $P \rightarrow M$, its Lie algebra is

$$\text{Lie } \mathcal{G} \cong \Omega^0(M, \mathfrak{g}). \quad (24)$$

An element of $\text{Lie } \mathcal{G}$ is a $\mathfrak{g}$ -valued field on spacetime — the same type of object as the ghost $\eta(x)$.²¹ Fix a tangent vector $\mathbb{X} \in T_A \mathcal{A}$. The connection evaluates on it to give $\varpi_A(\mathbb{X}) \in \text{Lie } \mathcal{G} \cong \Omega^0(M, \mathfrak{g})$: a $\mathfrak{g}$ -valued function on M returning, at each spacetime point x , the component of $\mathbb{X}$ that is pure gauge at x — i.e. the ghost field at x . The Grassmann character of η is the anticommutativity of one-forms on $\mathcal{A}$ under the wedge product, pulled back under (23): $\eta(x)\eta(y) = -\eta(y)\eta(x)$ is the statement that ϖ , viewed pointwise in spacetime, is a one-form with values in an anticommuting algebra. η as a field and ϖ as a one-form are the same object.

These identifications reproduce the BRST transformation laws of §2.2.²² For the gauge potential, the vertical derivative gives

$$\delta_V A_\mu = D_\mu \varpi, \quad (25)$$

which, under (23), is $sA_\mu = D_\mu \eta$. Read this way, sA is a one-form on $\mathcal{A}$, valued in $\text{Lie } \mathcal{G}$, whose evaluation on a tangent vector $\mathbb{X}$ returns the vertical component of $\mathbb{X}$ realised as a Lie-algebra-valued field on M ; the picture of a bosonic shift of A along its orbit is what treating η as a c-number gauge parameter yields. Verticality is the geometric statement that, evaluated on any $\mathbb{X} \in T_A \mathcal{A}$, sA lands in V_A .²³

For the ghost, the Cartan structure equation $\delta\varpi = \mathfrak{F} - \frac{1}{2}[\varpi, \varpi]$, restricted to vertical

²¹Strictly: $\text{Lie } \mathcal{G} \cong \Gamma(\text{ad } P)$, sections of the adjoint bundle $\text{ad } P = P \times_{\text{Ad}} \mathfrak{g}$. The isomorphism (24) is the trivialised form, valid globally when $P \rightarrow M$ is trivial and locally otherwise. I write $\Omega^0(M, \mathfrak{g})$ throughout, with $\Gamma(\text{ad } P)$ understood for non-trivial P .

²²One consequence of the type-clarification: the transposition from differential-form grading on $\mathcal{A}$ to particle-statistics grading on M — ghosts treated as fermionic for calculational purposes even though they are not fermions — is a feature of Berezin integration.

²³The derivation proceeds as follows: from the field-dependent gauge transformation $A^\beta = \beta^{-1}(A + d)\beta$, we compute δA^β and project onto vertical variations to get $\delta_V A^\beta = -D_{A^\beta}(\beta^{-1}\delta_V \beta)$. Identifying $\eta = \varpi$ with $\varpi := \beta^{-1}\delta_V \beta$ reproduces $sA = D_A \eta$ up to a sign reflecting left- versus right-invariant Maurer–Cartan form on $\mathcal{G}$; I fix the sign by Gomes and Riello (2017, §6).

arguments on which $\mathfrak{F}$ vanishes, gives²⁴

$$\delta_V \varpi = -\frac{1}{2}[\varpi, \varpi], \quad (26)$$

which under (23) is $s\eta = -\frac{1}{2}[\eta, \eta]$. Both BRST laws of (8) fall out of (22)–(23) together with the defining properties of a connection; neither extra structure nor flatness is required.

Nilpotency is automatic. The vertical distribution $\mathcal{V}$ is involutive — $[X^\#, Y^\#] = [X, Y]^\#$ on fundamental vertical fields — so the leafwise differential satisfies $\delta_V^2 = 0$. As a consistency check, applying δ_V to (26) and using the Jacobi identity gives

$$\delta_V^2 \varpi = -\frac{1}{2}\delta_V[\varpi, \varpi] = -[\delta_V \varpi, \varpi] = \frac{1}{2}[[\varpi, \varpi], \varpi] = 0, \quad (27)$$

so $s^2\eta = 0$. The same exercise on (25), using the graded Leibniz rule for δ_V on $D_A\varpi = d\varpi + [A, \varpi]$, returns $s^2A_\mu = 0$. BRST nilpotency is interpreted as the geometric fact that orbits are involutive distributions.

In summary: the BRST operator is the derivative of field-space forms along gauge orbits; the ghost is the connection one-form distinguishing orbit directions from their complement; and the Maurer–Cartan equation $s\eta = -\frac{1}{2}[\eta, \eta]$ is the vertical projection of the Cartan structure equation, satisfied by any principal connection on any principal bundle.

4.2 Why ϖ cannot be flat

A section and a flat connection are similar, but not quite the same. Clearly, a global section σ induces a flat connection: (right) translate the tangent spaces $T\sigma$ over each orbit by the group action (fn. 8), and the resulting copies of σ foliate $\mathcal{A}$ by horizontal leaves. Conversely, via Frobenius’ theorem, a flat connection integrates to such a foliation; and on a *simply connected* base each leaf is itself a global section. So on a simply connected base a global section exists if and only if a flat connection exists.²⁵

Three of Singer (1978)’s results bear directly on the geometric reading. Corollary 4 establishes that for $M = S^4$ (or S^3) and $G = \mathrm{SU}(N)$ with $N \geq 2$, no continuous

²⁴The bracket $[\varpi, \varpi]$ is itself a vertical 2-form: $[\varpi, \varpi](X, Y) = 2[\varpi(X), \varpi(Y)]$, which vanishes whenever either argument is horizontal, since ϖ does. So $\delta_V \varpi$ and $-\frac{1}{2}[\varpi, \varpi]$ are both forms on the orbit foliation, and the equation holds there.

²⁵The original Gribov (1978) ambiguity in Landau gauge is a different, related failure: $\partial^\mu A_\mu = 0$ intersects most orbits *more* than once. Even the first Gribov region (where $\mathcal{M}_A$ is positive) contains copies; the truly copy-free *fundamental modular region* is defined by absolute minimisation of $\|A\|^2$ along each orbit and has no closed-form description (Vandersickel and Zwanziger, 2012). The connection-theoretic obstruction $\mathfrak{F} \neq 0$ underlies both failures. Singer (1978) himself notes that paracompactness of $\mathcal{A}/G$ allows a partition-of-unity construction using locally-defined slices, but adds that “this partition of unity argument would be useful if the covering used could be made explicit” — a programme that does not appear to have been carried out.

global section $\sigma : \mathcal{A}/\mathcal{G} \rightarrow \mathcal{A}$ exists. Theorem 7 strengthens this: the bundle admits no flat connection at all — not just no flat ϖ_{sdw} , but no flat connection of any kind.²⁶ Theorem 9 gives the explicit curvature formula for the Singer–DeWitt connection (28); for non-Abelian G on S^4 that curvature is generically non-zero. Together, these constitute the *Gribov–Singer theorem*: in non-Abelian theories, gauge-fixing cannot give rise to a continuous global section, nor, equivalently, to a flat connection.²⁷

The derivation of (26) used only that $\mathfrak{F}$ vanishes on vertical arguments. The BRST algebra is the vertical projection of the Cartan structure equation, and that projection is insensitive to curvature. Within a single fibre, any reading that identifies the ghost with a flat object — the Maurer–Cartan form of $\mathcal{G}$ (Bonora and Cotta-Ramusino, 1983; Thierry-Mieg, 1980), or the Chevalley–Eilenberg generator of a flat action Lie algebroid (Dougherty, 2021; Dougherty and Read, 2026) — agrees with the connection reading. Each is the flat specialisation of (23): where ϖ is flat on an open $\mathcal{U} \subset \mathcal{A}$ its horizontal distribution integrates (§4.1), giving a local trivialisation $\mathcal{U} \simeq \pi(\mathcal{U}) \times \mathcal{G}$ on which ϖ is the pullback of the Maurer–Cartan form on $\mathcal{G}$. The Chevalley–Eilenberg complex $\wedge^\bullet(\text{Lie } \mathcal{G})^* \otimes C^\infty(\mathcal{A})$ is the algebraic reduction of ϖ in the same limit.

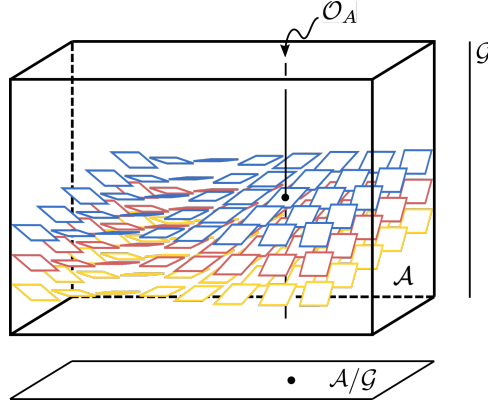


Figure 4: A non-integrable horizontal distribution. The horizontal planes (in colour) attached to points along the orbit $\mathcal{O}_A$ do not fit together into a foliation: parallel transport along a closed loop in $\mathcal{A}/\mathcal{G}$ returns the lift to a different point on the same fibre. The mismatch is the holonomy of ϖ ; infinitesimally it gives the curvature $\mathfrak{F} \neq 0$.

The standard explicit example of ϖ is the *Singer–DeWitt connection*, defined by an

²⁶The proof of Theorem 7 uses Theorem 6, which gives the topology of the base: $\mathcal{A}/\mathcal{G}$ is simply connected for $\text{SU}(N)$ with $N > 2$, and $\pi_1(\mathcal{A}/\mathcal{G}) = \mathbb{Z}_2$ for $\text{SU}(2)$ with even Pontryagin index. A flat connection on a simply-connected base would integrate to a section, contradicting Corollary 4. Theorem 7 therefore does independent work only for $\text{SU}(2)$, where a flat connection with non-trivial holonomy need not integrate to a section.

²⁷Babelon and Viallet (1981) give the Riemannian-geometric version of the obstruction — the Gribov horizon as the locus of conjugate points of the horizontal geodesic exponential map. Gomes and Riello (2017); Mitter and Viallet (1981) treat the bundle structure and standard-form connections in more detail.

orthogonality condition in a $\mathcal{G}$ -invariant supermetric on $\mathcal{A}$. Fix an ultralocal supermetric — the spacetime integral of contracted but undifferentiated tangent vectors, $\langle \mathbb{X}, \mathbb{Y} \rangle_A = \int_M \text{tr}(X_\mu Y^\mu) \text{vol}_g$, with no derivatives of $\mathbb{X}$ or $\mathbb{Y}$ appearing — and declare a tangent vector horizontal when it is orthogonal to V_A . Using the inner product on the algebra, we define the adjoint operator to D_A , namely $D_A^\dagger$. Then a tangent vector $\mathbb{X} \in T_A \mathcal{A}$ decomposes uniquely as $\mathbb{X} = D_A \xi + \mathbb{X}_\perp$ with $D_A^\dagger \mathbb{X}_\perp = 0$, and the Singer–DeWitt connection reads off the vertical coefficient:

$$\varpi_{\text{sdw}}(\mathbb{X}) = (D_A^\dagger D_A)^{-1} D_A^\dagger \mathbb{X} = \xi. \quad (28)$$

The construction requires $D_A^\dagger D_A$ to be invertible on $\text{Lie } \mathcal{G}$; its kernel consists of the reducibility parameters, which vanish on the irreducible stratum.

The obstruction has a local measure: the holonomy of ϖ . For a loop $\gamma \subset \mathcal{A}/\mathcal{G}$ bounding a surface $\mathcal{D}$, and a local section σ over $\mathcal{D}$,

$$\text{Hol}_\gamma(\varpi) = \mathcal{P} \exp \oint_\gamma \sigma^* \varpi = \mathcal{P} \exp \int_{\mathcal{D}} \sigma^* \mathfrak{F} \in \mathcal{G}, \quad (29)$$

with the second equality the non-Abelian Stokes theorem (Gomes et al., 2019).

For Abelian gauge theory the bracket on $\mathfrak{g}$ vanishes, so $\mathfrak{F} = \delta \varpi$ for any ϖ , and a closed (hence locally exact) one-form ϖ is flat. The Singer–DeWitt construction gives

$$\varpi_{\text{sdw}}(\mathbb{X}) = \Delta^{-1} \partial^\mu \delta A_\mu = \delta \Delta^{-1} \partial^\mu A_\mu, \quad (30)$$

(since Δ doesn’t depend on A) manifestly closed on $\mathcal{A}$, so $\mathfrak{F}[\varpi_{\text{sdw}}] = 0$ identically. Flatness in the Abelian case is not mandatory by abstract bundle theory — an Abelian connection on a principal bundle over spacetime can be curved — but on the field-space bundle the kinetic term and the group action are both field-independent, so the natural ultralocal construction returns a field-independent horizontal condition and the connection it defines is flat. A non-flat Abelian ϖ could be constructed by hand at the cost of giving up ultralocality.²⁸ The ghost-as-connection accordingly reduces, globally, to the ghost-as-Maurer–Cartan-form of Thierry-Mieg (1980), and the algebroid reading of Dougherty (2021) and Dougherty and Read (2026) captures it. Again, in the non-Abelian case the reduction fails and the algebroid registers only the vertical content of ϖ — the part the Faddeev–Popov calculus also uses. It does not register the horizontal complement where curvature, holonomy, and the Gribov obstruction live.²⁹

²⁸For $U(1)$ on $\mathbb{R}^3$ with rapid-fall-off boundary conditions, $\varpi_{\text{sdw}}(\mathbb{X}) = \Delta^{-1} \partial^i \delta A_i$ is the pure-gauge component of δA_i at each point. Promoting δA_i to a Grassmann-valued field-space one-form, the same formula returns the ghost $\eta(x)$ of Coulomb-gauge QED, and $sA_i = \partial_i \eta$, $s\eta = 0$ are the flat Maurer–Cartan equations of $U(1)$.

²⁹Leinaas and Olausen (1981) objected to identifications of ghosts with differential forms on a *finite-*

The connection ϖ also supplies a gauge-covariant field-space exterior derivative $\delta_H := \delta - \delta_V$, the field-space analogue of the spacetime covariant derivative $D_\mu = \partial_\mu + [A_\mu, \cdot]$. Quantities built from δ_H are $\mathcal{G}$ -covariant without selecting a section. Where there is no flat limit — non-Abelian gauge theory globally — only the covariant apparatus remains definable; see Gomes and Riello (2017, §4).

Parallel transport along ϖ is the field-theoretic form of Barbour’s best matching (Barbour, 2003; Barbour and Bertotti, 1982). Best matching compares two nearby configurations by shifting one along its orbit until the residual difference is minimised; for the ultralocal supermetric above, the minimising condition on $\mathbb{X} - D_A \xi$ is orthogonality to the orbit, $D_A^\dagger(\mathbb{X} - D_A \xi) = 0$, which is the horizontality condition of the Singer–DeWitt connection (28) (Gryb and Gomes, 2021). Horizontal lift is best matching iterated along a path, and the Gribov–Singer theorem bounds what the procedure can deliver: pairwise best matching exists everywhere, but $\mathfrak{F} \neq 0$ measures its failure to cohere into a path-independent global alignment.

5 Objections and comparisons

The reading developed here treats the ghost as the connection on field space. Rival readings exist — BRST as Slavnov–Taylor cohomology (Piguet and Sorella, 1995), as flat algebroid structure (Dougherty, 2021; Dougherty and Read, 2026), or, within BV/BFV, as one component of a shifted cotangent bundle alongside antifields (Batalin and Vilkovisky, 1981; Cattaneo et al., 2014).

Coexistence with these rivals is not by itself a recommendation. In my view the connection reading earns its place by economy: it adds no degrees of freedom and no extended phase space, and derives rigidity and the Maurer–Cartan equation from the single identification $\eta = \varpi$. But the comparison invites an objection, which the two subsections below take up from opposite directions.

5.1 Conventionality

The counterpart relation is relative to a choice of ϖ , and nothing in the physics strictly speaking privileges one. Coulomb and Lorenz alignments differ, and the Singer–DeWitt connection is fixed only once a supermetric is given.

dimensional principal bundle on grounds of nilpotency: products of forms of degree above the manifold’s dimension vanish, while products of ghosts do not. Their fix was to take $\mathcal{G}$ as an infinite-dimensional Lie group in its own right. The present framework extends this: the home is not only the gauge group but the principal bundle of field space, which carries non-trivial topology on top of infinite-dimensional structure.

That much is part of the view. The objection is the inference drawn from this consideration: that because the algebroid reading is choice-invariant, the connection is dispensable. For it is natural to hold that, because the algebroid reading is choice-invariant (the Chevalley–Eilenberg complex is the same whichever ϖ one picks), it already captures everything needed, and the connection belongs to the representational scheme rather than the theory.

However, although a particular ϖ depends on a choice, the fact that some ϖ must be fixed does not. Locally on field space, every quantisation that sums over distinct orbits needs at least a choice of section; globally, by Gribov–Singer, each must fix a non-flat connection rather than a section (§4.2). Charts on a manifold behave similarly: each computation needs one, none is privileged and none (usually) covers the whole. Moreover, connections on $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$ form an affine space whose differences are tensorial — horizontal, equivariant Lie $\mathcal{G}$ -valued one-forms — and gauge-fixing independence of physical amplitudes can be verified by their constancy across this space. The algebraic reading records that invariance cohomologically, in the BRST-exactness of the gauge-fixing sector (Appendix A); the geometric reading displays the space over which the invariance holds.

Thus I hold that no particular counterpart relation is a commitment of the theory. The commitment is to the class of such structures: some connection must be fixed, usually by pragmatic criteria, and in the non-Abelian case it cannot be flat.

5.2 The Vilkovisky–DeWitt programme: connection as primitive

The conventionality objection can also be met head-on, by privileging a ϖ . The *Vilkovisky–DeWitt programme* (DeWitt, 2003; Pawłowski, 2003; Vilkovisky, 1984) does this, treating ϖ as a primitive of quantisation. A $\mathcal{G}$ -invariant supermetric on $\mathcal{A}$, read off the Lagrangian, singles out a preferred ϖ ; its splitting (20) supplies a horizontal projector at each A , and the geodesic normal fields of the supermetric coordinatise fluctuations around a background by gauge-invariant variables. The path integral is defined over those horizontal variables, and the effective action is gauge-invariant by construction. The standard Faddeev–Popov integral, by contrast, uses ϖ only as the linearised horizontal complement at the background, where section and connection coincide infinitesimally; Vilkovisky–DeWitt promotes ϖ to the global structure that defines the integration domain.

A preferred supermetric would answer conventionality by removing the choice, but the thesis here needs no preferred ϖ . Thus, whether ϖ should be promoted to a primitive of quantisation is a question I leave open.

6 Conclusion

The paper opened with two puzzles; both dissolve on the same identification. *Puzzle (1)* asked what classical structure on $\mathcal{A}$ the ghosts encode. The answer offered here, extending the classical-content line of Dougherty (2021) and Dougherty and Read (2026): the ghost is the connection ϖ on the principal bundle $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$, and the structure it encodes is the vertical/horizontal splitting of field space at each A — equivalently, the cross-orbit pairing that gauge-fixed quantisation, dressing constructions, and counterfactual comparison all use; but that pairing is discarded by the bare quotient. *Puzzle (2)* asked how BRST can survive gauge-fixing, given that its action on A looks like an infinitesimal gauge transformation. It survives because it was never the target: BRST is the rigid, vertical exterior derivative on $\mathcal{A}$ — one nilpotent differential, not the $\lambda(x)$ ’s worth of gauge transformations one is free to vary — and ϖ ’s equivariance preserves the splitting it defines.

The operational reading (§3) and the geometric reading (§4) converge. Where ϖ is flat, its horizontal distribution integrates to local sections, and the connection-based counterpart relation reduces to the section-based one. Where Gribov–Singer forbids flatness (§4.2), cross-orbit alignment survives as parallel transport along ϖ : path-dependent, with holonomy measured by $\mathfrak{F}$. Best matching is the same operation under another name (§4.2): pairwise alignment exists everywhere; the curvature measures its failure to cohere into a single global scheme.

The picture bears on the debate over *sophistication* about symmetry (Dewar, 2017) — broadly, the policy of treating symmetry-related configurations as identical rather than as a redundancy to be eliminated.³⁰ Gomes (2026) distinguishes the contexts in which sophistication suffices for the treatment of symmetry from those in which it does not; the moral is that tasks requiring cross-orbit comparison — quantisation, local observables, regional gluing — need alignment across orbits on top of the within-orbit identification that sophistication supplies. The functional connection ϖ provides that alignment; rigid BRST protects it in the gauge-fixed path integral. BRST, in this light, marks the limit of sophistication: identifying configurations within an orbit leaves the pairing between orbits under-described, and quantisation (often) tacitly uses such a pairing. The pairing is choice-relative; the need for it is not (§5.1).

Three questions remain open. *Non-perturbatively*: whether the global non-flatness of ϖ has observable consequences — confinement, the Gribov horizon, the non-perturbative path-integral measure — is the question the Vilkovisky–DeWitt programme (DeWitt,

³⁰The position has precursors in Saunders (2003), Belot (2003), and Caulton (2015); Dewar (2017) gives the most explicit modern formulation. March and Read (2025) contains a fuller bibliography.

2003; Donkin and Pawłowski, 2012; Pawłowski, 2003; Vilkovisky, 1984) is best placed to address. *Anomalies*: the Chevalley–Eilenberg reading places them in the Lie-algebra cohomology of $\mathcal{G}$; in the non-flat setting the relevant cohomology should be that of ϖ itself, and the story when $\mathfrak{F} \neq 0$ is, to my knowledge, not worked out.³¹ *Boundaries*: ϖ already does classical work in the covariant symplectic treatment of bounded regions (Carrozza and Höhn, 2022; Donnelly and Freidel, 2016; Gomes et al., 2019); its BRST counterpart, and the role of $\mathfrak{F}$ in edge-mode dressings, remain open.

None of this displaces the Faddeev–Popov calculus. Feynman rules and Grassmann integration are unaffected; the geometric reading supplies an account of the objects the rules are rules *for*, and of why the rules have the shape they do. BRST is the algebraic record of the fibre structure of field space, not a residue of gauge symmetry that gauge-fixing failed to remove.

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A The ghost and the gauge-fixing doublet

The identifications (22)–(23) concern the ghost. They do not say that every Faddeev–Popov auxiliary field is a further component of the same field-space connection. We need to distinguish two structures: the bundle geometry of $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$ that is fixed by the gauge action alone, and the implementation of a chosen gauge-fixing functional, which is a downstream choice. The ghost belongs to the first structure; the antighost $\bar{\eta}$ and the Nakanishi–Lautrup field B belong to the second.

The vertical/horizontal split at each A , supplied by ϖ , reads off vertical motion: $\varpi_A(\mathbb{X}) \in \text{Lie } \mathcal{G}$ is the gauge parameter of the vertical part of $\mathbb{X} \in T_A \mathcal{A}$. Choosing ϖ is choosing the horizontal complement, and when ϖ is flat this is locally equivalent to choosing a gauge-fixing functional F ; what distinguishes the two presentations is global behaviour, since the defining equations (18)–(19) are intrinsic to the bundle and survive cases (§4.2) where no F can be globally chosen. The Faddeev–Popov operator, by contrast, is built from F directly. In Lorenz gauge $W = \Omega^0(M, \mathfrak{g})$ and $F(A) = \partial^\mu A_\mu$; in Coulomb gauge $W = \Omega^0(\mathbb{R}^3, \mathfrak{g})$ and $F(A) = \partial_i A^i$. In each case $\mathcal{M}_A$ is the derivative of F along the

³¹The Dai–Freed treatment of global anomalies already sits on the geometric side: it locates the anomaly in the determinant-line bundle over $\mathcal{A}/\mathcal{G}$, whose Bismut–Freed curvature measures the local anomaly much as $\mathfrak{F}$ measures the obstruction to a flat gauge-fixing (Dai and Freed, 1994).

vertical directions:

$$\mathcal{M}_A = dF_A \circ \#_A : \text{Lie } \mathcal{G} \rightarrow W, \quad (31)$$

where $\#_A$ is the fundamental-field map of §4.1. So $\mathcal{M}_A$ measures how the chosen slice Σ_F cuts the orbit through A , not how the bundle is put together.

The antighost is naturally dual to the gauge-fixing condition rather than to the ghost. When $A \in \Sigma_F$ and dF_A is surjective, the dual map $dF_A^* : W^* \rightarrow T_A^* \mathcal{A}$ identifies W^* with the conormal directions to Σ_F ; $\bar{\eta}_A$ is an odd section of ΠW^* , where Π denotes parity reversal of the fibres. Equivalently, $\bar{\eta}_A$ is an odd conormal variable for the slice. The Nakanishi–Lautrup field B is its even partner, $B_A \in W^*$. The two are paired through $\mathcal{M}_A$ alone, via the Berezin identity (5); without F no such pairing exists.

How $\mathcal{M}_A$ enters the BRST cohomology is best seen through the gauge-fixing fermion. Define

$$\Psi_F = \int_M \bar{\eta} \cdot \left(F[A] - \frac{\xi}{2} B \right). \quad (32)$$

Applying the BRST operator via the laws (8) gives

$$s\Psi_F = \int_M \left(iB \cdot F[A] - \frac{i\xi}{2} B \cdot B - \bar{\eta} \cdot \mathcal{M}_A \eta \right). \quad (33)$$

The first two terms reproduce S_{GF} of (7) with B replaced by iB — the imaginary-axis integration convention of §2.1 made explicit — and eliminating B by its algebraic equation of motion gives the standard quadratic gauge-fixing term; the last term is precisely S_{FP} . The whole gauge-fixing sector of $\tilde{S}$ is BRST-exact, $S_{\text{GF}} + S_{\text{FP}} = s\Psi_F$, hence cohomologically trivial. That η and $\bar{\eta}$ meet only through this exact expression, with $\mathcal{M}_A$ as their sole point of contact, expresses the geometric fact that the antighost belongs to the slice.

The doublet $s\bar{\eta} = iB$, $sB = 0$ is BRST-contractible so it contributes no cohomology. It is because it is contractible that physical amplitudes are independent of F . On the geometric side the same fact takes a different form. ϖ 's curvature is intrinsic to $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{G}$, while $\mathcal{M}_A$ becomes degenerate exactly where the chosen Σ_F ceases to cut the orbit transversally: namely, where some non-zero vertical vector $D_A \lambda$ lies tangent to the slice,

$$dF_A(D_A \lambda) = 0, \quad (34)$$

so that $\mathcal{M}_A \lambda = 0$. The Gribov horizon is the locus of that degeneration; in the Faddeev–Popov formula it appears as a zero of the ghost–antighost pairing, and §4.2 reads it off the global non-flatness of ϖ . The Gribov horizon is a degeneracy of the chosen gauge-fixing map F , while ϖ 's curvature records the gauge-fixing-independent obstruction to replacing connection-based transport by a global flat section.

References

- Babelon, O. and Viallet, C. M. (1979). The Geometric Interpretation of the Faddeev–Popov Determinant. *Physics Letters B* 85, 246–248.
- Babelon, O. and Viallet, C. M. (1981). The Riemannian Geometry of the Configuration Space of Gauge Theories. *Communications in Mathematical Physics* 81, 515–525.
- Barbour, J. (2003). Scale-Invariant Gravity: Particle Dynamics. *Classical and Quantum Gravity* 20, 1543–1570.
- Barbour, J. and Bertotti, B. (1982). Mach’s Principle and the Structure of Dynamical Theories. *Proceedings of the Royal Society A* 382, 295–306.
- Batalin, I. A. and Vilkovisky, G. A. (1981). Gauge Algebra and Quantization. *Physics Letters B* 102, 27–31.
- Becchi, C., Rouet, A. and Stora, R. (1976). Renormalization of Gauge Theories. *Annals of Physics* 98, 287–321.
- Belot, G. (2003). Notes on Symmetries. In K. Brading and E. Castellani (eds.), *Symmetries in Physics: Philosophical Reflections*, 393–412. Cambridge University Press.
- Bertlmann, R. A. (2000). *Anomalies in Quantum Field Theory*. Oxford University Press.
- Bonora, L. and Cotta-Ramusino, P. (1983). Some Remarks on BRS Transformations, Anomalies and the Cohomology of the Lie Algebra of the Group of Gauge Transformations. *Communications in Mathematical Physics* 87, 589–603.
- Brunetti, R., Fredenhagen, K. and Verch, R. (2003). The Generally Covariant Locality Principle—A New Paradigm for Local Quantum Field Theory. *Communications in Mathematical Physics* 237(1–2), 31–68.
- Carrozza, S. and Höhn, P. A. (2022). Edge Modes as Reference Frames and Boundary Actions from Post-Selection. *Journal of High Energy Physics* 2022(2).
- Cattaneo, A. S., Mnev, P. and Reshetikhin, N. (2014). Classical BV Theories on Manifolds with Boundary. *Communications in Mathematical Physics* 332, 535–603.
- Caulton, A. (2015). The Role of Symmetry in the Interpretation of Physical Theories. *Studies in History and Philosophy of Modern Physics* 52, 153–162.
- Dai, X. and Freed, D. S. (1994). η -Invariants and Determinant Lines. *Journal of Mathematical Physics* 35(10), 5155–5194. arXiv:hep-th/9405012.

- Dewar, N. (2017). Sophistication about Symmetries. *British Journal for the Philosophy of Science* 70(2), 485–521.
- DeWitt, B. S. (2003). *The Global Approach to Quantum Field Theory*. Oxford University Press.
- Diez, T. and Rudolph, G. (2019). Slice Theorem and Orbit Type Stratification in Infinite Dimensions. *Differential Geometry and its Applications* 65, 176–211.
- Donkin, I. and Pawłowski, J. M. (2012). The Phase Diagram of Quantum Gravity from Diffeomorphism-Invariant RG Flows. arXiv:1203.4207.
- Donnelly, W. and Freidel, L. (2016). Local Subsystems in Gauge Theory and Gravity. *Journal of High Energy Physics* 2016(9), 102.
- Dougherty, J. (2021). I Ain’t Afraid of No Ghost. *Studies in History and Philosophy of Science* 88, 70–84.
- Dougherty, J. and Read, J. (2026). Conceptual Aspects of Non-Positive Quantization. Manuscript.
- Ebin, D. (1970). The Manifold of Riemannian Metrics. In *Proceedings of Symposia in Pure Mathematics* 15, 11–40. American Mathematical Society.
- Faddeev, L. D. and Popov, V. N. (1967). Feynman Diagrams for the Yang–Mills Field. *Physics Letters B* 25, 29–30.
- Goeller, C., Höhn, P. A. and Kirklin, J. (2022). Diffeomorphism-Invariant Observables and Dynamical Frames in Gravity: Reconciling Bulk Locality with General Covariance. arXiv:2206.01193.
- Gomes, H. (2025). Representational Schemes for Theories with Symmetry. *Synthese* 206(1).
- Gomes, H. (2026). Background-Relative Sophistication. Manuscript, arXiv:2602.13708.
- Gomes, H. and Riello, A. (2017). The Observer’s Ghost: Notes on a Field Space Connection. *Journal of High Energy Physics* 2017(5), 017.
- Gomes, H., Hopfmüller, F. and Riello, A. (2019). A Unified Geometric Framework for Boundary Charges and Dressings: Non-Abelian Theory and Matter. *Nuclear Physics B* 941, 249–315.
- Gribov, V. N. (1978). Quantization of Nonabelian Gauge Theories. *Nuclear Physics B* 139, 1–19.

- Gryb, S. and Gomes, H. (2021). Turbo-Charging Relationalism. *Studies in History and Philosophy of Modern Physics*.
- Healey, R. (2007). *Gauging What's Real: The Conceptual Foundations of Gauge Theories*. Oxford University Press.
- Isenberg, J. and Marsden, J. E. (1982). A Slice Theorem for the Space of Solutions of Einstein's Equations. *Physics Reports* 89, 179–222.
- Kment, B. (2012). Haecceitism, Chance, and Counterfactuals. *Philosophical Review* 121(4), 573–609.
- Kondracki, W. and Rogulski, J. (1983). On the Stratification of the Orbit Space for the Action of Automorphisms on Connections. Polish Academy of Sciences, Institute of Mathematics.
- Leinaas, J. M. and Olaussen, K. (1981). Ghosts and Geometry. CERN Preprint TH-3061.
- March, E. and Read, J. (2025). What is this sophistication thing anyway? Manuscript.
- Mitter, P. K. and Viallet, C. M. (1981). On the Bundle of Connections and the Gauge Orbit Manifold in Yang–Mills Theory. *Communications in Mathematical Physics* 79, 457–472.
- Pawlowski, J. M. (2003). Geometrical Effective Action and Wilsonian Flows. arXiv:hep-th/0310018.
- Peskin, M. E. and Schroeder, D. V. (1995). *An Introduction to Quantum Field Theory*. Addison-Wesley.
- Piguet, O. and Sorella, S. P. (1995). *Algebraic Renormalization: Perturbative Renormalization, Symmetries and Anomalies*. Springer.
- Roberts, A. (2026). Speaking for Haecceitists. *Philosophers' Imprint* 26(10).
- Saunders, S. (2003). Indiscernibles, General Covariance, and Other Symmetries: The Case for Non-Reductive Relationalism. In A. Ashtekar, R. S. Cohen, D. Howard, J. Renn, S. Sarkar and A. Shimony (eds.), *Revisiting the Foundations of Relativistic Physics: Festschrift in Honour of John Stachel*. Kluwer.
- Singer, I. M. (1978). Some Remarks on the Gribov Ambiguity. *Communications in Mathematical Physics* 60, 7–12.

- Thierry-Mieg, J. (1980). Geometrical Reinterpretation of Faddeev–Popov Ghost Particles and BRS Transformations. *Journal of Mathematical Physics* 21, 2834–2838.
- Tyutin, I. V. (1975). Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism. Lebedev Physics Institute preprint 39.
- Vandersickel, N. and Zwanziger, D. (2012). The Gribov Problem and QCD Dynamics. *Physics Reports* 520, 175–251.
- Vilkovisky, G. A. (1984). The Gospel According to DeWitt. In S. M. Christensen (ed.), *Quantum Theory of Gravity: Essays in Honor of the 60th Birthday of Bryce S. DeWitt*. Adam Hilger.